MULTIMEDIA **UNIVERSITY**

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FINAL EXAMINATION

TRIMESTER 3, 2016 / 2017

PEM0046 – INTRODUCTION TO PROBABILITY AND STATISTICS
(All sections / Groups)

29 MAY 2017
2:30 p.m. – 4:30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of **FIVE** printed pages including the cover page.

2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the answer booklet provided. All necessary working steps **MUST** be shown.

4. Statistical table is provided.

Answer ALL questions [100 marks].

QUESTION 1 [25 marks]

a.   Figure Q1a shows a dart board area where two concentric circles with radii 2 cm and 4 cm respectively, drawn on a circular target with radius 6 cm. A player scores 10, 5 or 3 points for hitting the target inside the smallest circle, middle annular region and outer annular region respectively. The probability that a dart will hit a given region is proportional to the area of the region.
Let X denotes the number of points scored on a single throw of a dart.

   i.   Find the probability distribution for X.   [8 marks]
   $\left[\text{Hint : Area of a circle} = \pi r^2 \right]$

   ii. Calculate the mean and standard deviation for this probability distribution.
                                                                                              [4 marks]

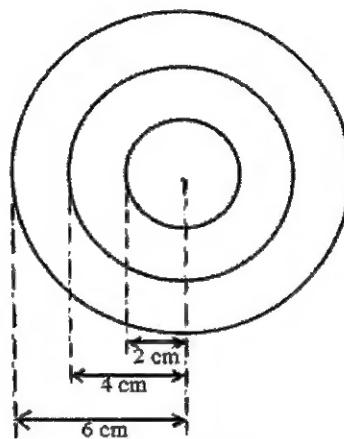Figure Q1a

b.   A food manufacturer is aware that there is a slight variation in the weight for each box of their products. From production data in the past, it has been determined that the variable weight (in grams) can be described as

$$f(x) = \begin{cases} \dfrac{2}{3} & , \quad 15.23 \le x \le 16.73 \\ 0 & , \quad elsewhere \end{cases}$$

   i.   Verify that this is a valid density function.   [3 marks]

   ii. Determine the cumulative distribution function, $F(X)$.   [4 marks]

   iii. Find the probability that the weight is not more than 16.3 grams.   [3 marks]

   iv. The company desires that the weight exceeding 16.5 grams be an extremely rare occurrence. What is the probability that this rare occurrence does actually occur?   [3 marks]

Continued...

QUESTION 2 [25 marks]

a.   In a group of n people, the average number who uses the public transportation is 3 and the variance is 2.1. Assuming that the number of people who uses the public transportation follows the binomial distribution,

      i.   find the values for n and p, where p is the probability that a person is using public transportation when chosen at random.   [4 marks]

      ii.   find the probability that exactly three people in the group using public transportation.   [2 marks]

      iii.   find the probability that between 2 and 6 people in the group NOT using public transportation.   [4 marks]

b.   The quality of computer disks is measured by sending the disk through a certifier which counts the number of missing pulses. If a certain brand of computer disks averages 0.2 missing pulse per disk,

      i.   find the probability that the next inspected disk will have no missing pulse.
                                                                                                [1 mark]

      ii.   find the probability that the next disk inspected will have more than one missing pulse.   [2 marks]

      iii.   find the probability that neither of the next two disks inspected will contain any missing pulse.   [2 marks]

c.   An orange juice producer buys all his oranges from a large orange farm that has one variety of orange. The amount of juice squeezed from each of these oranges is approximately normal distribution with a mean of 4.70 ounces and a standard deviation of 0.4 ounce.

      i.   What is the probability that the amount of juice squeezed from a randomly selected orange will contain between 4.70 and 5.00 ounces?   [3 marks]

      ii.   What is the probability that the amount of juice squeezed from a randomly selected orange will be less than the population mean by 0.5 ounce or more?
                                                                                                [3 marks]

      iii.   Find the amount of juice squeezed so that 77% of the oranges will be above this value.   [4 marks]

Continued...

QUESTION 3 [25 marks]

a. Consider a large population with the standard deviation, $\sigma = 2.10$.

    i. Find the standard deviation of the sample mean, $\sigma_{\bar{x}}$ for a sample size of $n = 30$ and $n = 60$.   [3 marks]

    ii. As the sample size, n increases, explain how does the values of $\sigma_{\bar{x}}$ change?   [1 mark]

b. A population consists of the six numbers: **2, 4, 6, 8, 10, 6**.

Consider all possible samples of **size two** which can be drawn without replacement from this population.

    i. Find the mean, μ and standard deviation, σ of the population.   [5 marks]
    [Leave your answer in three decimal points]

    ii. List all the possible samples of size two (without replacement) from this population, and hence determine the sample mean.   [5 marks]

    iii. Tabulate the sampling distribution of the sample mean, $P(\bar{x})$ in a table.   [3 marks]

    iv. If a sample of two numbers (8 and 10) is selected from this population, what is the sampling error?   [2 marks]

c. The height of nine-year-old males is normally distributed with mean, $\mu = 133.3 \, cm$ and standard deviation, $\sigma = 5 \, cm$. If a random sample of size 40 is selected,

    i. what is the probability that the sample mean is between 131 cm and 135 cm?   [3 marks]

    ii. if 90% of the nine-year-old males in the sample have the mean height less than $\bar{x}$, what is the value of $\bar{x}$?   [3 marks]

QUESTION 4 [25 marks]

Table Q4 shows the final scores in algebra and calculus obtained by 8 students selected at random from a large group of students.

Algebra, x	30	40	35	55	82	90	15	70
Calculus, y	15	31	23	50	72	80	5	62

Table Q4

a. Find the least square regression line fitting the data, using x as the independent variable.
[13 marks]

b. Plot the scatter diagram and the least square regression line on the same graph.
Describe the relationship between the two variables.   [5 marks]

c. If a student receives a score of 80 in algebra, what is her expected score in calculus?
[2 marks]

d. Calculate the linear correlation coefficient and give your comment on the value obtained.   [5 marks]

(Note: For Question 4, leave all your answers in 3 decimal points)

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